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Complete Positivity and Subdynamics in Quantum Field Theory†

B. Vacchini[1]

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The relevance that the property of complete positivity has had in the determination of quantum structures is briefly reviewed, together with recent applications to neutron optics and quantum Brownian motion. A possible useful application and generalization of this property to the description of macroscopic systems in quantum mechanics is discussed on the basis of recent work on the derivation of subdynamics in the Heisenberg picture of slowly varying degrees of freedom inside nonrelativistic quantum field theory.

1. INTRODUCTION

Even though a thorough understanding of quantum mechanics (QM) is still far away and many different readings and interpretations of QM coexist in the scientific community, major progress has been made in the study and determination of quantum structures, both from a logical and mathematical point of view. The logical studies on QM originated in the seminal paper of Birkhoff and von Neumann (1936) and have by now reached important results, mainly thanks to the basic notions of effect and effect algebras (for a recent review see Dalla Chiara and Giuntini, n.d.). On the physical and mathematical side it is hardly feasible to do justice to the full inventory of mathematical tools and properties that have been introduced and understood to be relevant to the realm of QM, and especially quantum measurement theory. In fact, all studies concerning quantum structures aim at a better understanding of the foundations of QM and therefore often address either directly or indirectly the subject of measurement theory.

†This paper is dedicated to the memory of Prof. Gottfried T. Rüttimann.

[1]Dipartimento di Fisica dell'Università di Milano and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, I-20133, Milan, Italy; e-mail: bassano.vacchini@mi.infn.it.

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As far as this paper is concerned, we are mainly interested in notions and tools that have grown out of the so-called operational approach to QM (for a recent review see Busch *et al.*, 1995), whose physical origin is to be traced back to the original work of Ludwig and coworkers on the foundations of QM (Ludwig, 1983). As a matter of fact, the work of Ludwig, which was originally intended to obtain a reconstruction of the Hilbert space structure of QM based on a statistical formulation of the theory relying on a classical, objective description of preparation and registration apparatuses, apart from giving a fundamental contribution to the foundations of QM, has also somehow "incidentally" led to the introduction of concepts, such as those of effect and operation, that are by now very useful in applications of QM, such as quantum optics and quantum information. In particular, the notions of effect and operation, together with more general and refined mathematical tools derived from these, such as those of coexistent observable, effect-valued measure, and instrument, have made the very formulation of continuous measurement feasible (Srinivas and Davies, 1981; Barchielli *et al.*, 1983), perhaps one of the major achievements within quantum measurement theory. This result, though somehow to be expected on the basis of experimental evidence—consider, for example, the shelving effect (Dehmelt, 1990)—was certainly not obvious in the early days of QM, and provides some evidence substantiating the conjecture of Ludwig about the possibility of founding QM on macroscopic systems to be objectively described in a suitable trajectory space.

In the sequel we will deal in particular with the notion of complete positivity (CP), whose relevance to the realm of QM was first realized by Kraus (1971) and Lindblad (1976). This property has recently played an outstanding role in the study of open quantum systems, both at a fundamental as well as at a phenomenological level, and we will give a few important examples in which it has actually led to the determination of specific quantum structures. We will then argue how CP, and in particular a generalized, mathematically less stringent version of this property, might play a role in the determination of subdynamics inside nonrelativistic quantum field theory. This will be done on the basis of recent work carried out in the fields of neutron optics (Lanz and Vacchini, 1997a,b) and quantum Brownian motion (Lanz and Vacchini, in preparation), as well as a recently outlined approach for the description of the dynamics of slowly varying degrees of freedom within a macroscopic system (Lanz *et al.*, 1997; Lanz and Vacchini, 1998). This should shed some light on possible useful extensions of the property of CP from one-particle QM to the realm of quantum field theory applied to many-body systems.

2. COMPLETE POSITIVITY

Let us now briefly introduce the definition of CP. The most general representation of the preparation of a physical system described in a Hilbert space $\mathcal{H}$ is given by a statistical operator, that is, an operator in the space $\mathcal{TC}(\mathcal{H})$ of trace class operators on $\mathcal{H}$, positive and with trace equal to one. In particular, we call $\mathcal{K}(\mathcal{H})$ the convex set of statistical operators

$$\mathcal{K}(\mathcal{H}) = \{\hat{\rho} \in \mathcal{TC}(\mathcal{H}) | \hat{\rho} \geq 0, \text{Tr}\hat{\rho} = 1\}$$

Consider now a mapping $\mathcal{U}$ defined on the space of trace class operators into itself

$$\mathcal{U}: \quad \mathcal{TC}(\mathcal{H}) \to \mathcal{TC}(\mathcal{H})$$

possibly corresponding to a Schrödinger-picture description on the states. We say that the map $\mathcal{U}$ is completely positive, or equivalently has the property of complete positivity (Kraus, 1983; Alicki and Lendi, 1987), if and only if the adjoint map $\mathcal{U}'$ acting on the space $\mathcal{B}(\mathcal{H})$ of bounded linear operators, dual to $\mathcal{TC}(\mathcal{H})$,

$$\mathcal{U}': \quad \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$$

and therefore corresponding to a Heisenberg-picture description in terms of observables, satisfies the inequality

$$\sum_{i,j=1}^{n} \langle \psi_i | \mathcal{U}'(\hat{B}_i^\dagger \hat{B}_j) | \psi_j \rangle \geq 0$$
$$\forall n \in \mathbf{N}, \quad \forall \{\psi_i\} \in \mathcal{H}, \quad \forall \{\hat{B}_i\} \in \mathcal{B}(\mathcal{H}) \tag{2.1}$$

For $n = 1$ one recovers the usual notion of positivity, while for bigger n this is actually a nontrivial requirement.

It is immediately seen that if $\mathcal{U}'$ has the factorization property

$$\mathcal{U}'(\hat{A}^\dagger \hat{B}) = [\mathcal{U}'(\hat{A})]^\dagger \, \mathcal{U}'(\hat{B}), \qquad \forall \hat{A}, \hat{B} \in \mathcal{B}(\mathcal{H})$$

then it is CP, so that any unitary evolution is CP. In this sense one can see CP as a property that is worth retaining when shifting from the unitary dynamics for closed systems to a more general dynamics for the description of open systems. In fact, the general physical argument for the introduction of CP is the following. Consider a system $\mathcal{S}_1$ described in $\mathcal{H}_1$, whose dynamics is given by the family of mappings

$$\mathcal{U}: \quad \mathcal{TC}(\mathcal{H}_1) \to \mathcal{TC}(\mathcal{H}_1)$$

and an n-level system $\mathcal{S}_2$ described in $\mathcal{H}_2 = \mathbf{C}^n$, whose dynamics can be neglected, so that $\hat{H}_2 = 0$. Because the two systems do not interact, the map $\tilde{\mathcal{U}}$ describing their joint evolution

$$\tilde{\mathcal{U}}: \quad \mathcal{TC}(\mathcal{H}_1 \otimes \mathbf{C}^n) \to \mathcal{TC}(\mathcal{H}_1 \otimes \mathbf{C}^n)$$

will be simply given by the tensor product $\tilde{\mathcal{U}} = \mathcal{U} \otimes \mathbf{1}$. But the dynamical map $\tilde{\mathcal{U}}$ must of course be positive and this is equivalent to the requirement that $\mathcal{U}$ be CP.

The property of CP has already shown to be particularly relevant in the determination of quantum structures, and in the sequel we will give two important examples in this connection. Let us consider first the notion of operation, which is the basic tool for the description of changes experienced by a physical system. An operation $\mathcal{F}$ is a positive linear map acting on the space of trace class operators and sending statistical operators in positive operators with trace less than or equal to one,

$$\mathcal{F}: \quad \mathcal{T}\mathcal{C}(\mathcal{H}) \rightarrow \mathcal{T}\mathcal{C}(\mathcal{H}), \qquad 0 \leq \mathrm{Tr}\mathcal{F}(\hat{\rho}) \leq 1, \quad \forall \hat{\rho} \in \mathcal{K}(\mathcal{H})$$

Operations describe the repreparations of a statistical collection based on some measurement outcome, and the connection between such mappings and CP was first considered by Kraus (1983) and Hellwig (1995). The requirement of CP, according to the Stinespring representation theorem, determines the general structure of such mappings to be

$$\mathcal{F}(\hat{T}) = \sum_{k \in K} \hat{A}_k^\dagger \hat{T} \hat{A}_k, \qquad \forall \hat{T} \in \mathcal{T}\mathcal{C}(\mathcal{H}),$$

$$K \subset \mathbf{N}, \quad 0 \leq \sum_{k \in K} \hat{A}_k^\dagger \hat{A}_k = \hat{F} \leq 1$$

where $\hat{F}$ is the effect associated to the operation, even though not uniquely specifying it. Let us note that the notion of operation, previously considered only in the studies of fundamental nature about QM and quantum measurement theory, is now being used by a much broader physical community thanks to the applications in quantum optics and more recently quantum communication and quantum information theory. For example, CP trace-preserving operations are by now taken as the standard definition of quantum channels (Schumacher, 1996).

The other example comes from the field of quantum dynamical semigroups (Alicki and Lendi, 1987), used for the description of the irreversible dynamics of open quantum systems, typically the reduced dynamics of systems interacting with an external system, such as a heat bath or a measuring instrument. In Heisenberg picture, quantum dynamical semigroups are given by collections of positive mappings

$$\mathcal{U}'_t: \quad \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H}), \qquad t \geq 0, \quad \mathcal{U}'_0 \mathbf{1} = \mathbf{1}$$

which satisfy the semigroup composition property

$$\mathcal{U}'_s \mathcal{U}'_t = \mathcal{U}'_{s+t}, \qquad s, t \geq 0$$

and which are normal. Under these conditions a generally unbounded generator $\mathcal{L}'$ defined on an ultraweakly dense domain exists such that

$$\frac{d}{dt} \mathcal{U}'_t \hat{B} = \mathcal{L}' \mathcal{U}'_t \hat{B}$$

for all $\hat{B}$ in the domain. If one further asks the semigroup to be norm continuous, so that the generator is a bounded map, it can be shown, as has been done by Lindblad (1976), that CP determines the general expression for the generator to be of the form

$$\mathcal{L}'\hat{B} = \frac{i}{\hbar}[\hat{H}, \hat{B}] - \frac{1}{2}\left\{\sum_j \hat{V}_j \hat{V}_j^\dagger, \hat{B}\right\} + \sum_j \hat{V}_j^\dagger \hat{B} \hat{V}_j \hat{V}_j, \sum_j \hat{V}_j \hat{V}_j^\dagger \in \mathcal{B}(\mathcal{H}),$$
$$\hat{H}_j = \hat{H}_j^\dagger \in \mathcal{B}(\mathcal{H})$$

This structure of the master equation, possibly allowing for unbounded operators or even quantum fields, appears in many applications in very different fields of physics and is often taken as a starting point for phenomenological approaches. It accounts for a non-Hamiltonian dynamics and has been extensively used in the formulation of continuous measurement theory and especially in quantum optics.

3. SUBDYNAMICS IN QUANTUM FIELD THEORY

Inspired by Ludwig's work, we take the attitude according to which any macroscopic physical system is actually defined by the preparation procedure which separates it from the rest of the world. Such systems are to be described in terms of interacting and confined quantum fields, whose choice depends on the considered description level. In fact, realistic confinement and isolation can only be considered with reference to a coarse graining of the time scale, which allows us to consider a breakup of the correlations with the environment and to replace the actual physical walls by suitably idealized boundary conditions. For the description of the system on the given time scale we look for the subdynamics of a subset of variables which are slowly varying on this time scale. Contrary to older attempts at the derivation of a master equation for the reduced dynamics of the statistical operator, we work in the Heisenberg picture on a restricted set of observables whose choice depends on the particular features of the system and of the preparation procedure. In considering the subdynamics of these quantities, on the basis of the experienced gained with one-particle QM, we will ask for the property of CP, a viewpoint shared by Streater (1995). Using the field-theoretic formalism of second quantization, observables have the general expression

$$\hat{A} = \sum_{\substack{h_1 \ldots h_n \\ k_1 \ldots k_n}} a_{h_n}^\dagger \ldots a_{h_1}^\dagger \mathcal{A}(h_n, \ldots, h_1, k_1, \ldots, k_2) a_{k_1} \ldots a_{k_n}$$

so that, recalling (2.1), this typical structure in terms of a block of annihilation operators and a block of creation operators is a natural candidate for the requirement of CP. Moreover, typical slow variables will be positive densities of conserved quantities, such as mass and energy, which are easily expressed in a field formalism and whose positivity has to be preserved throughout the evolution.

Before going considering some concrete examples of this formal scheme, let us briefly mention the possible relevance of this approach to the foundations of QM. A general formulation of the theory of macroscopic systems in terms of a non-Hamiltonian irreversible dynamics for a selected set of observables could be the starting point for their description inside continuous measurement theory, thus possibly recovering a classical, objective description, but objectively described macroscopic systems build the basis on which Ludwig founded his remarkable approach to the foundations of QM.

The simplest application of the proposed formal scheme consists in the description of a microsystem interacting with a macroscopic system supposed to be at equilibrium, typically a particle interacting with matter. In this case the particle constitutes the slow degree of freedom with respect to the fast relaxation time of the macroscopic system and a Markov approximation for the particle's dynamics should hold, provided the two time scales are separated. We here briefly sketch the main points in the formalism and calculation [for details see Lanz and Vacchini (1997a)]. In second quantization the Hamiltonian reads

$$\hat{H} = \hat{H}_0 + \hat{H}_m + \hat{V}, \qquad \hat{H}_0 = \sum_f E_f a_f^\dagger a_f, \qquad [a_f, a_g^\dagger]_\mp = \delta_{fg}$$

The whole system is described in $\mathcal{H}$, while the Hilbert space for the microsystem $\mathcal{H}^{(1)}$ is spanned by the energy eigenstates u_f, so that a_f is the destruction operator for the microsystem, whose statistics is left unprejudiced, in the state u_f. $\hat{H}_m$ describes matter and $\hat{V}$ is the interaction potential. Since we are interested in the description of a single particle, we take a statistical operator of the form

$$\hat{\rho} = \sum_{gf} a_g^\dagger \hat{\rho}_m a_f \rho_{gf}^{(1)}$$

where $\hat{\rho}_m$ describes matter and $\rho_{gf}^{(1)}$ is a positive matrix with trace equal to one. In order to extract the subdynamics of the particle, we consider field observables of the form $\hat{A} = \sum_{h,k} a_h^\dagger A_{hk}^{(1)} a_k$ and exploit the reduction formula

$$\mathrm{Tr}_{\mathcal{H}}(\hat{A}\hat{\rho}) = \mathrm{Tr}_{\mathcal{H}^{(1)}}(\hat{A}^{(1)}\hat{\rho}^{(1)})$$

where (1) denotes operators in $\mathcal{H}^{(1)}$. What we have to evaluate is the time evolution, in the Heisenberg picture, of bilinear structures of field operators $a_h^\dagger\, a_k$ on a suitable time scale-much longer than the microphysical interaction time τ_0. To do this, we exploit a superoperator formalism, thus considering maps acting on the algebra of creation and destruction operators, for example,

$$\mathcal{H}'_0(a_h^\dagger a_k) = \frac{i}{\hbar}[\hat{H}_0 + \hat{H}_{\rm m}, a_h^\dagger a_k] = \frac{i}{\hbar}(E_h - E_k)a_h^\dagger a_k$$

and exploit techniques of scattering theory. We thus work in a Markov approximation considering slow variables on the given time scale, so that the quasi-diagonality condition $\hbar/|E_h - E_k| \geq t \gg \tau_0$ should be satisfied. We also suppose suitable smoothness properties of the T-matrix, so that there are no bound states. As a result we obtain the following structure for the evolution mapping on a time t which is small with respect to the particle's dynamics, though much larger than the relaxation time of the macrosystem:

$$\mathcal{U}'(t)(a_h^\dagger a_k) = a_h^\dagger a_k + t\mathcal{L}' a_h^\dagger a_k$$

where the generator restricted to this typical bilinear structure of field operators in the quasi-diagonal case is given by:

$$\mathcal{L}'(a_h^\dagger a_k) = \frac{i}{\hbar}[\hat{H}_0 + \hat{V}^{[1]}, a_h^\dagger a_k] - \frac{1}{\hbar}\{[\hat{\Gamma}^{[1]}, a_h^\dagger]a_k - a_h^\dagger[\hat{\Gamma}^{[1]}, a_k]\}$$
$$+ \frac{1}{\hbar}\sum_\lambda \hat{R}_{h\lambda}^{[1]\dagger}\hat{R}_{k\lambda}^{[1]}$$

the index [1] denoting one-particle operators, and $\hat{V}^{[1]}$ and $\hat{\Gamma}^{[1]}$ being linked respectively to the self-adjoint and anti-self-adjoint parts of the T-matrix. Let us note that due to the presence of the minus sign the term between curly brackets cannot be rewritten as a simple commutator. CP of the mapping $\mathcal{U}'(t)$ restricted to these simple bilinear field structures

$$\sum_{i,j=1}^n \langle\psi_i|\mathcal{U}'(t)\left(\sum_{hk} a_h^\dagger\langle h|\hat{\mathrm{B}}_i^\dagger\hat{\mathrm{B}}_j|k\rangle a_k\right)|\psi_j\rangle \geq 0$$

can be seen from the decomposition which holds true for an infinitesimal positive time dt,

$$a_h^\dagger a_k + dt\,\mathcal{L}'(a_h^\dagger a_k) = \left\{a_h + \frac{i}{\hbar}dt[\hat{H}_0 + \hat{V}^{[1]}, a_h] - \frac{dt}{\hbar}[\hat{\Gamma}^{[1]}, a_h]\right\}^\dagger$$
$$\times\left\{a_k + \frac{i}{\hbar}dt[\hat{H}_0 + \hat{V}^{[1]}, a_k] - \frac{dt}{\hbar}[\hat{\Gamma}^{[1]}, a_k]\right\}$$
$$+ \frac{dt}{\hbar}\sum_\lambda \hat{R}_{h\lambda}^{[1]\dagger}\hat{R}_{k\lambda}^{[1]}$$

One can also check that particle number conservations holds, so that $\mathscr{L}'(\hat{N}) = 0$, where $\hat{N} = \Sigma_f \, a_f^\dagger a_f$. In order to consider the microsystem's degrees of freedom, we take a partial trace over matter

$$\frac{d}{dt}\,\rho^{(1)}_{kh} = \mathrm{Tr}_{\mathscr{H}}(\mathscr{L}'(a_h^\dagger a_k)\hat{\rho}_{\mathrm{m}})$$

and obtain the following master equation of the Lindblad form for the subdynamics of the particle, thus automatically ensuring CP:

$$\frac{d}{dt}\,\hat{\rho}^{(1)} = -\frac{i}{\hbar}\,[\hat{H}_0^{(1)} + \hat{V}^{(1)},\, \hat{\rho}^{(1)}] - \frac{1}{\hbar}\,\{\hat{\Gamma}^{(1)},\, \hat{\rho}^{(1)}\} + \frac{1}{\hbar}\sum_{\xi\lambda} \hat{L}^{(1)}_{\lambda\xi}\hat{\rho}^{(1)}\hat{l}^{(1)\dagger}_{\lambda\xi}$$

where $\hat{V}^{(1)}$ and $\hat{L}^{(1)}$ are still linked to the self-adjoint and anti-self-adjoint parts of the T-matrix averaged over matter, and particle number conservation implies $\hat{L}^{(1)} = 1/2\, \Sigma_{\xi\lambda}\, \hat{L}^{(1)}_{\lambda\xi}\hat{L}^{(1)}_{\lambda\xi}$. This master equation is well suited to describe both coherent and incoherent behavior. Apart from a commutator term analogous to a Liouville–von Neumann equation, it has an anticommutator term, which might be obtained by introducing a complex potential in the Schrödinger equation, and a mixture term which is only characteristic of the formalism of the statistical operator. It will here be applied to two examples.

In the first case we consider the mainly coherent interaction of thermal neutrons with homogeneous samples, so-called neutron optics, relevant for the description of neutron interferometry experiments [for references and details see Lanz and Vacchini (1997b)]. The phenomenological Ansatz used for the description of neutron matter interaction is the Fermi pseudopotential

$$\hat{T} = \frac{2\pi\hbar^2}{m}\, b \int d^3\mathbf{r}\; \psi^\dagger(\mathbf{r})\; \delta^3(\hat{x} - \mathbf{r})\psi(\mathbf{r})$$

which is a local potential parametrized by the coherent scattering length b. Using this Ansatz and leaving out the incoherent, dissipative terms, we obtain the usual wavelike description of the interaction in terms of a refractive index $n \simeq [1 - (\lambda^2/2\pi)bn_o]$ depending on the density n_o of particles in the medium. The dissipative contributions instead can be expressed in terms of the dynamic structure function of the medium

$$S(\mathbf{q}, \mathrm{w}) = \frac{1}{2\pi N}\int dt \int d^3\mathbf{x}\; e^{-i(\mathrm{w}t - \mathbf{g}\cdot\mathbf{x})} \int d^3\mathbf{y}\; \langle \hat{N}(\mathbf{y})\hat{N}(\mathbf{x} + \mathbf{y}, t)\rangle$$

where $\hbar\mathrm{w}$ and $\hbar\mathbf{q}$ denote energy and momentum transfer, and $\hat{N}(\mathbf{x}) = \psi^\dagger(\mathbf{x})\psi(\mathbf{x})$ local densities. These terms lead to an imaginary correction to the optical potential

$$\hat{\mathrm{U}} = \frac{2\pi\hbar^2}{m} n_o \left[b - i \frac{b^2}{4\pi} \frac{p_0}{\hbar} \int d\Omega_q \, S(\mathbf{q}) \right]$$

which takes diffuse scattering into account, thus straightforwardly recovering a result previously obtained through multiple scattering theory. The incoherent contribution instead accounts for fullfilment of the optical theorem and is possibly responsible for loss of coherence in interferometric experiments.

The other application concerns so-called quantum Brownian motion (Vacchini, in preparation). In this case one considers the dissipative dynamics of a particle interacting with a gas by two-body collisions. Since we are interested in the local dissipative behavior, we neglect the influence of the actual boundary conditions, and suppose that the system may be considered locally homogeneous within a very good approximation, thus analyzing the interaction in momentum space. Under the assumption of small momentum transfers, the balance between the anticommutator and incoherent term leads to a quantum generalization of the Fokker–Planck equation having a CP structure (Diósi, 1995), while derivations starting at a fundamental level usually miss positivity of the time evolution (Ambegaokar, 1991), let alone CP. The equation reads

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{\mathrm{H}}_0 + \hat{\mathrm{V}}, \hat{\rho}] - \frac{1}{\hbar^2} D_{pp} \sum_{i=1}^{3} [\hat{\mathrm{x}}_i, [\hat{\mathrm{x}}_i, \hat{\rho}]] - D_{qq} \sum_{i=1}^{3} [\hat{\mathrm{p}}_i, [\hat{\mathrm{p}}_i, \hat{\rho}]] - \frac{i}{\hbar} D_{qp} \sum_{i=1}^{3} [\hat{\mathrm{x}}_i, \{\hat{\mathrm{p}}_i, \hat{\rho}\}] \tag{3.2}$$

where $\hat{\mathrm{V}}$ is a mean-field potential, D_{pp} is expressed in terms of the scattering cross section, and

$$D_{qq} = \left(\frac{1}{4MkT}\right)^2 D_{pp}, \qquad D_{qp} = \frac{1}{2MkT} D_{pp}$$

M being the mass of the particle. Equation (3.2) actually has a Lindblad structure (Barchielli, 1983), as can be seen by introducing the generators $\hat{L}_i = \hat{x}_i + i(\hbar/2MkT)\hat{p}_i$, thus yielding

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar} [\hat{\mathrm{H}}_0 + \hat{\mathrm{V}}, \hat{\rho}] - \frac{i}{\hbar} \frac{D_{pp}}{4MkT} \sum_{i=1}^{3} [\{\hat{\mathrm{x}}_i, \hat{\mathrm{p}}_i\}, \hat{\rho}] - 2 \frac{D_{pp}}{\hbar^2} \sum_{i=1}^{3} \left[\frac{1}{2} \{\hat{\mathrm{L}}_i^\dagger \hat{\mathrm{L}}_i, \hat{\rho}\} - \hat{\mathrm{L}}_i \hat{\rho} \hat{\mathrm{L}}_i^\dagger \right]$$

We will now briefly sketch how the proposed formal scheme may be applied to the simplest cases of systems having many degrees of freedom [see Lanz *et al.* (1997) and Lanz and Vacchini (1998) for further details]. Slow variables inside a many-body system, characteristically corresponding to densities of conserved charges, will have the form

$$\hat{A}(\boldsymbol{\xi}) \sum_{hk} a_h^\dagger A_{hk}(\boldsymbol{\xi}) a_k, \qquad \hat{B}(\boldsymbol{\xi}) \sum_{\substack{h_1 h_2 \\ k_1 k_2}} a_{h2}^\dagger a_{h1}^\dagger B_{h_2 h_1 k_1 k_2}(\boldsymbol{\xi}) a_{k1} a_{k2}$$

and similarly for quantities involving a higher number of field operators, where the couples of indexes h_i, k_i are linked by a quasi-diagonality condition due to the slow variability. One is therefore faced with evaluating in the Heisenberg picture on a time t much longer than collision times, but still short with respect to the variation time of slow observables,

$$\mathcal{U}'(t)(a_h^\dagger\, a_k) = e^{i/h\hat{H}t} a_h^\dagger a_k e^{-i/h\hat{H}t}$$

or, more generally,

$$\mathcal{U}'(t)(a_{h_n}^\dagger \ldots a_{h_1}^\dagger a_{k_1} \ldots a_{k_n}) = e^{i/h\hat{H}t} a_{h_n}^\dagger \ldots a_{h_1}^\dagger a_{k_1} \ldots a_{k_n} e^{-(i/h)\hat{H}t}$$

Similarly, as before, we ask that this map $\mathcal{U}'(t)$ satisfy a less stringent version of CP; in fact, we ask CP only when it is applied on these structures of blocks of field operators in the sufficiently diagonal case, i.e., when acting on the relevant, slowly varying observables. Analogously as before, calculations have been put forward using a superoperator formalism, considering a self-interacting Schrödinger field, i.e., a gas of particles interacting through a short-range potential, and working in a one-mode approximation, so that three-particle collisions are neglected. The result for the generator is formally the same as before,

$$\mathcal{L}'(a_h^\dagger a_k) = \frac{i}{\hbar}[\hat{H}_0 + \hat{V}^{[2]}, a_h^\dagger a_k] - \frac{1}{\hbar}\{[\hat{\Gamma}^{[2]}, a_h^\dagger] a_k - a_h^\dagger [\hat{\Gamma}^{[2]}, a_k]\}$$
$$+ \frac{1}{\hbar}\sum_{\lambda} \hat{R}_{h\lambda}^{[2]\dagger} \hat{R}_{k\lambda}^{[2]}$$

but [2] now denotes two-particle operators, and statistical corrections are properly accounted for in the structure of the T-matrix. A slight generalization of this result holds in the case of $2n$ operators.

We therefore hope to have shown new examples of useful application of the property of CP, indicating how the restriction of the requirement of CP to a suitable subset of slowly varying quantities might be a guiding principle in the determination of subdynamics inside nonrelativistic quantum field theory.

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